Homework assignment 1 – Mathematical Statistics 2023
Hand in your own solutions at the start of the tutorial on September 14.
Consider a continuous random variable X with the following probability density function:

$$f(x) = \begin{cases} k(2x - x^2) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a normalization constant.

a) Determine the value of k such that $f(x)$ is a valid probability density function.

b) Compute the cumulative distribution function (c.d.f.) of X.

c) Determine the median of X.

d) Calculate $P(0.5 \leq X \leq 1.5)$.

e) Show that X and $Y = 2 - X$ are identically distributed.

Let Z be another random variable defined as $Z = X^2$.

f) Determine the probability density function of Z.

g) Calculate the expected value $E(Z)$.

h) Compute the variance of Z.

Grading:

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**Total**
Points	1	1	1	1	1	1	2	2	**10**