

Test 1 for Probability Theory
(Module Signals and Uncertainty, 202001342)
Monday March 6, 2023, 8.45 - 10.15 hour.

This test consists of 4 problems.
 Use proper notation and motivate all answers.
 Using a calculator is *not* allowed.

(iii) $P(S)$

1. Consider a probability space (S, P) .
 - a. The first two axioms of Kolmogorov are given by (i) $P(E) \geq 0$ for any event $E \subset S$ and (ii) $P(S) = 1$. Give the third axiom (iii), and prove that (i)-(iii) imply that $P(E) \leq 1$ for any event $E \subset S$. *If E_1, E_2, E_3, \dots are n.e. $\Rightarrow P(\cup E_i) = P(E_1) + P(E_2) + \dots$*
 - b. Now assume S is a finite set and let P be given by the Laplace definition. Show that P satisfies (i) and (ii).

2. We choose a person at random from the Dutch population and measure the length X of the person. Assume the probabilities of the person being male or female are both 0.5. The length of a male person is assumed to be normally distributed with expectation μ and standard deviation σ , while the length of a woman is normally distributed with expectation ν and standard deviation τ . (Hint: define a suitable event to indicate whether the chosen person is male or female.)

Normally = $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

 - a. Give an expression for $P(X \leq x)$ in terms of Φ and the given parameters, where Φ is the distribution function of a standard normal random variable Z . *$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$*
 - b. Give an expression (again in terms of Φ and the given parameters) for the probability that a woman was chosen if we already know that the length of the person was not more than x . *$P(W|X \leq x) = \frac{P(W, X \leq x)}{P(X \leq x)}$*
 - c. What is the distribution of $2Y - 10$ where Y is the weight of a male person?
length

3. A second-hand car dealer has 20 cars for sale, five of which are reliable (and 15 not). We pick two cars at random for a test drive. Let X be the number of reliable cars in our pick.
 - a. Give the range S_X and the probability mass function of X .
 - b. Sketch the form of the distribution function of X , with relevant values on the axes. (If you did not answer a. above, then assume that X takes values 0, 1 and 2 with probabilities 21/39, 16/39 and 2/39 respectively).
 - c. Can the distribution of X be reasonably approximated by a Binomial distribution with $n = 2$ and $p = 1/4$? Argue why (not); no computations are needed.

4. The random variable X has an exponential distribution with parameter 3.
 - a. Determine the probability $P(6X > X^2 + 8)$.
 - b. Determine the density of the random variable Y given by $Y = e^{-X}$.
 - c. Determine $E[e^{-X}]$. *$\int_{-\infty}^{\infty} e^{-x} 3e^{-3x} dx = [$*

Norm: (grade = total/3 + 1)

1		2			3			4			Total
a	b	a	b	c	a	b	c	a	b	c	
3	2	3	2	2	3	2	2	3	3	2	27
✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	