

**Test Linear Structures 2.**  
**Applied Mathematics, 2022-1B: Structures and Systems**

This exam consists of 10 problems which are divided into two parts:

**Grasple (digital):** 8 problems

**Open Questions (written):** 2 problems.

**Grasple (13:45-15:15)**

Enter your answers in Grasple in the required form. Follow the instructions precisely.

All exercises are constructed to be solvable within the given time.

If you feel like your solution method requires too many steps, it might be useful to consider a different solution method.

For the statements, you choose one of three options: true (T), false (F), or no answer (N).

For each correct T or F you will receive (partial) points.

One incorrect T or F results in zero points for that entire question.

**Total score for Grasple:** 40 points.

**Required score:** 20 points.

**Open Questions (15:30-17:00)**

Write the solutions following the four steps.

**Step 1.** State the important information and summarize the problem.

**Step 2.** Devise a plan.

**Step 3.** Execute the plan.

**Step 4.** Analyze your solution and the answer.

Think, for example, of the following questions:

- Can you interpret the solution and/or the result intuitively?
- Does the solution/result make sense in special cases?
- Which role each condition played in the solution?
- Could you relax some assumptions of the problem?
- Is the problem related to other problems or results?

**Total score for Open Questions:** 40 points.

Steps 1+2: 40%, Step 3: 40%, Step 4: 20%.

**Required score:** 20 points.

**Grade:**  $1+9(\text{number of points})/80$ .

A (graphical) calculator is not needed and is **not allowed** at the exam.

**PART 2: Open questions.**

9. [20pt] Consider a square matrix  $A$ . Recall that  $O$  denotes the zero-matrix. For both of the following statements, either prove it, or give a counterexample.
- (a) If  $A^k = O$  for some  $k \in \mathbb{N}$ , then  $0$  is the only eigenvalue of  $A$ .
  - (b) If  $0$  is the only eigenvalue of  $A$ , then  $A^k = O$  for some  $k \in \mathbb{N}$ .
10. [20pt] Consider  $\mathbb{C}^n$  with standard inner product. Consider  $n \times n$  matrix  $A$  with (possibly) complex coefficients.  
Prove that  $A$  is self-adjoint if and only if  $L_A$  is self-adjoint.