PART 2: Open questions.

For each exercise, you can combine parts a) and b) in steps 1 and 4. That is, you only need to write 1 problem statement and 1 reflection for exercise 9. You also need only one of each for exercise 10.

[20pt] Consider a square matrix A. Let $B = P^{-1}AP$ for some invertible matrix P.

1. Prove that for any eigenvector v of A: $P^{-1}v$ is an eigenvector of B.
2. Prove that if A is diagonalizable, then B is diagonalizable.

[20pt] Consider finite-dimensional real inner product space V.
For each of the following statements, either prove it, or give a counterexample. If you give a counterexample, then also prove that it is a counterexample.

1. For any self-adjoint linear operator T on V: There exists a self-adjoint operator U on V for which $U^2 = T$.
2. For any self-adjoint linear operator T on V: There exists a self-adjoint operator U on V for which $U^3 = T$.