Test Linear Structures 2.
Applied Mathematics, 2022-1B: Structures and Systems

This exam consists of 10 problems which are divided into two parts:
Grasple (digital): 8 problems
Open Questions (written): 2 problems.

Grasple (This morning)
Enter your answers in Grasple in the required form. Follow the instructions precisely.
For the statements, you choose one of three options: true (T), false (F), or no answer (N).
For each correct T or F you will receive (partial) points.
One incorrect T or F results in zero points for that entire question.
Total score for Grasple: 40 points.
Required score: 20 points.

Open Questions (Now)
Write the solutions following the four steps.
Step 1. State the important information and summarize the problem.
Step 2. Devise a plan.
Step 3. Execute the plan.
Step 4. Analyze your solution and the answer.
Think, for example, of the following questions:
– Can you interpret the solution and/or the result intuitively?
– Does the solution/result make sense in special cases?
– Which role each condition played in the solution?
– Could you relax some assumptions of the problem?
– Is the problem related to other problems or results?
Total score for Open Questions: 40 points.
Steps 1+2: 40%, Step 3: 40%, Step 4: 20%.
Required score: 20 points.
Grade: 1+9(number of points)/80.

A (graphical) calculator is not needed and is **not allowed** at the exam.

PART 2: Open questions.

9. [20pt] Consider a linear operator T on a vector space V. Let $\boldsymbol{u}, \boldsymbol{v}$ denote eigenvectors of T from different eigenspaces. Let $\boldsymbol{u}+\boldsymbol{v} \in W$, where W is a T-invariant subspace of V.
Prove that $\boldsymbol{u} \in W$ and that $\boldsymbol{v} \in W$.

10. [20pt]

(a) Let T be a normal linear operator on a finite-dimensional complex inner product space V.
Prove that there exists a normal operator U for which $U^2 = T$.

(b) Let T be a unitary linear operator on a finite-dimensional complex inner product space V.
Prove that there exists a unitary operator U for which $U^2 = T$.