PART 2: Open questions.

10. [20pt] Let V be a finite-dimensional vector space over F, and suppose $T : V \to V$ is a linear transformation.

a) Prove that if $T(x) = kx$ for a scalar k (i.e. T is a scalar multiple of the identity transformation) then $ST = TS$ for all linear transformations $S : V \to V$.

b) Prove that if there exists an $x \in V$ such that $T(x) \neq kx$ for any scalar k, then there exists a linear transformation $S : V \to V$ such that $ST \neq TS$.
Hint: for any $v \in V$, $T(v)$ can be written as a linear combination of basis vectors.

11. [20pt] Let $T : V \to W$ be a one-to-one linear transformation and let $\{u_1, u_2, \ldots, u_p\} \subseteq V$ be a linear independent set. Prove that the set $\{T(u_1), T(u_2), \ldots, T(u_p)\} \subseteq W$ is linearly independent and $p \leq \min\{\dim(V), \dim(W)\}$.