Resit Exam: Analysis-2 (202200237), MOD-02-AM: Structures and Systems

Instructors: Felix Schwenninger and Carlos Pérez Arancibia

Date/Time: 16-April-2024, 8:45 – 11:45

- Closed book exam! May use one single-sided handwritten A4-paper.
- All answers must be motivated, including the answers of Section C.
- Answers for Section A could use the four steps.
- Section Grade: $\dfrac{\text{Obtained score}}{\text{Total points}} \times 9 + 1$ (rounded off to one decimal place)
- Course Grade: $0.4 \times \text{Grade\_Section\_A} + 0.6 \times \text{Grade\_Section\_C}$ (see Assessment Policy for details)
- Good Luck!

Section C:

Total Points : 30

1. (a) Evaluate the integral: [5]

$$\int_0^1 x \ln x dx.$$

(b) Express the following sum as a Riemann integral and evaluate it: [2+3]

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{\sqrt{n^4 + (nk)^2}}.$$

2. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

(a) Find the limit of $f(x,y)$ as $(x,y) \to (0,0)$ and determine whether $f : \mathbb{R}^2 \to \mathbb{R}$ is continuous. [2+2]

(b) Compute the partial derivatives $\partial f(x,y)/\partial x$ and $\partial f(x,y)/\partial y$ for any $(x,y) \in \mathbb{R}^2$. [2+2]

(c) Verify that $f(x,y)$ is NOT differentiable at $(0,0)$. [2]

3. Evaluate the integral [10]

$$\iint_D \frac{1}{(x^2+y^2)^2} dxdy,$$

where the region $D \subset \mathbb{R}^2$ is determined by the conditions $x^2 + y^2 \leq 1$ and $x + y \geq 1$.

Section A:

Total Points : 20

4. Consider the following power series:

$$S(x) = \sum_{k=2}^{\infty} k x^{k-2}.$$

(a) Find the radius of convergence and the largest set $D \subset \mathbb{R}$ where the series $S(x)$, $x \in D$, converges. [3+3]

(b) Find a closed form expression for $S : D \to \mathbb{R}$. [4]

5. Let $c \in \mathbb{R}$ and $f_n : [0, \infty) \to \mathbb{R}$ be defined by

$$f_n(x) = n^c \cdot x \cdot \mathrm{e}^{-\frac{x}{n}}$$

for $n \in \mathbb{N}$.

(a) Determine for which c the sequence f_n converges pointwise on $[0, \infty)$ and find the limit function f. [3+3]

(b) For which values c does the sequence converge uniformly on $[0, \infty)$? [4]